Micromechanical Prediction of the Effective Behavior of Fully Coupled Electro-Magneto-Thermo-Elastic Multiphase Composites

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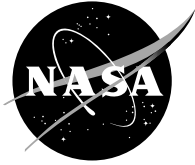
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Abstract

The micromechanical generalized method of cells model is employed for the prediction of the effective moduli of electro-magneto-thermo-elastic composites. These include the effective elastic, piezoelectric, piezomagnetic, dielectric, magnetic permeability, electromagnetic coupling moduli, as well as the effective thermal expansion coefficients and the associated pyroelectric and pyromagnetic constants. Results are given for fibrous and periodically bilaminated composites.

1. Introduction

The method of cells and its generalization, referred to as the generalized method of cells (GMC), is an approximate analytical micromechanical model which is capable of predicting the overall behavior of continuous and discontinuous (short-fiber and particulate) multiphase composites from the knowledge of the properties of the individual phases and their volume fractions. A review of the method has been recently updated in Aboudi (1996), where critical assessments of the method and its application by various researchers were outlined. As documented, many types of composites (e.g. thermoelastic, viscoelastic, nonlinear elastic and

viscoplastic) have been analyzed by the method, and the reliability of the predictions were demonstrated under many circumstances. For a detailed explanation of the two-dimensional case with four subcells see the recent textbook by Herakovich (1997).

To provide a framework for GMC, a code referred to as MAC/GMC has been developed at NASA Glenn Research Center. Through MAC/GMC, various thermal, mechanical and thermomechanical load histories can be imposed, different integration algorithms can be selected, many different fiber architectures can be utilized and a variety of fiber and matrix constitutive models are available (see Arnold et al. (1999), a user guide for version 3.0).

In an attempt to incorporate intelligent materials into GMC, this method has been extended in a recent paper (Aboudi, 1998) and applied for the prediction of the effective elastic, piezoelectric, dielectric, pyroelectric and thermal expansion coefficients of multiphase composites with embedded piezoelectric phases. The predictive capability of GMC was assessed in this case by comparison with results of Dunn and Taya (1993) and Dunn (1993), and very good agreements were obtained.

Piezoelectric materials exhibit interactions among mechanical, electric and thermal effects. A detailed account on piezoelectric materials, actuators and their applications has been given in a recent monograph by Uchino (1997). A generalization of the analysis of electro-mechanical interaction effects would be the incorporation of magnetic field. Such a generalization would enable the analysis of composite systems that involve the interaction of electro-magneto-thermo-mechanical effects. The analytical modeling of such composites provides the opportunity to study the effect of controlling and altering the response of composite structures that consist of composite materials the phases of which are, in general, electro-magneto-thermo-elastic. Piezoelectric and piezomagnetic composites would be obtained merely as special cases of the general theory. A first step in developing such a hybrid analytical approach has been presented by Carman et al (1995) where a concentric cylinder model has been employed. More recently, Li and Dunn (1998) presented a micromechanical methodology for the analysis of electro-magneto-thermo-elastic composites, and the Mori-Tanaka mean field approach has been applied to generate the effective moduli of composites that consist of piezoelectric and piezomagnetic phases.

In the present paper, the fully coupled electro-magneto-thermo-elastic constitutive equa-

tions are incorporated into GMC in order to establish the various types of effective moduli of **multiphase** intelligent composites whose constituents, in general, behave according to these equations. Piezoelectric and piezomagnetic phases can be obtained as special cases from these constitutive laws. Several comparisons of the predicted results with those provided by Li and Dunn (1998) for **two-phase** composites are given.

2. Basic Equations

The constitutive equations that govern the interaction of elastic, electric, magnetic and thermal fields in a electro-magneto-thermo-elastic medium relate the stresses σ_{ij}, strains ϵ_{ij}, electric field E_i, magnetic field H_i and temperature deviation ΔT (from a reference temperature T_0) as follows (see Li and Dunn (1998), for example):

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} - e_{ijk}E_k - q_{ijk}H_k - \Lambda_{ij}\Delta T \qquad i,j,k,l = 1,...,4 \qquad (1)$$

where C_{ijkl}, e_{ijk}, q_{ijk} and Λ_{ij} denote the fourth order elastic stiffness tensor, the third order piezoelectric tensor, the third order piezomagnetic tensor, and the second order thermal stress tensor of the material, respectively.

In addition, the **electric displacement vector**, D_i, is expressed in terms of the strain, electric field, magnetic field and temperature in the form

$$D_i = e_{ikl}\epsilon_{kl} + \kappa_{ik}E_k + a_{ik}H_k - p_i\Delta T \qquad (2)$$

where κ_{ik}, a_{ik}, and p_i are the second order dielectric tensor, the second order magnetoelectric coefficient tensor, and the pyroelectric vector, respectively.

Finally, the **magnetic flux density vector**, B_i, is given in terms of the mechanical, electric, magnetic fields and temperature by

$$B_i = q_{ikl}\epsilon_{kl} + a_{ik}E_k + \mu_{ik}H_k - m_i\Delta T \qquad (3)$$

where μ_{ik} and m_i are the second order magnetic permeability tensor and the pyromagnetic vector, respectively. These tensors satisfy certain symmetry conditions that are given by Nye (1957).

Let the vectors $\mathbf{X}$ and $\mathbf{Y}$ be defined as follows:

$$\mathbf{X} = [\epsilon_{11}, \epsilon_{22}, \epsilon_{33}, 2\epsilon_{23}, 2\epsilon_{13}, 2\epsilon_{12}, -E_1, -E_2, -E_3, -H_1, -H_2, -H_3] \tag{4}$$

$$\mathbf{Y} = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}, D_1, D_2, D_3, B_1, B_2, B_3] \tag{5}$$

Consequently, equations (1)-(3) can be written in the following compact matrix form:

$$\mathbf{Y} = \mathbf{Z}\,\mathbf{X} - \mathbf{\Gamma}\Delta T \tag{6}$$

where the square $12th$ order symmetric matrix of coefficients $\mathbf{Z}$ has the following form

$$\mathbf{Z} = \begin{bmatrix} \mathbf{C} & \mathbf{e^t} & \mathbf{q^t} \\ \mathbf{e} & \mathbf{-\kappa} & -\mathbf{a} \\ \mathbf{q} & -\mathbf{a} & \mathbf{-\mu} \end{bmatrix} \tag{7}$$

and

$$\mathbf{\Gamma} = \left\{ \begin{matrix} \mathbf{\Lambda} \\ \mathbf{p} \\ \mathbf{m} \end{matrix} \right\} \tag{8}$$

In equation (7), the square matrix $\mathbf{C}$ of the 6th-order represents the 4th order stiffness tensor, $\mathbf{e^t}$ denotes the transpose of the rectangular 3 by 6 matrix $\mathbf{e}$ that represents the corresponding third order piezoelectric tensor, $\mathbf{q^t}$ denotes the transpose of the rectangular 3 by 6 matrix $\mathbf{q}$ that represents the corresponding third order piezomagnetic tensor, κ is a square matrix of order 3 that corresponds to the dielectric tensor, $\mathbf{a}$ is a square matrix of the 3rd order that represents the magnetoelectric coefficients, and μ is a square matrix of the 3rd order that represents the magnetic permeability tensor. Finally, the 6th order vector $\mathbf{\Lambda}$, and the two 3rd order vectors $\mathbf{p}$ and $\mathbf{m}$ in Eqn. (8) represent the thermal stresses, pyroelectric and pyromagnetic coefficients, respectively.

3. Micromechanical Analysis

Consider a multiphase composite material in which some or all phases are modeled as electro-magneto-thermo-elastic materials. It is assumed that the composite possesses a periodic structure such that a repeating cell can be defined. In Fig. 1, such a repeating cell is shown which consists of $N_\alpha N_\beta N_\gamma$ rectangular parallelepiped subcells. The volume of each subcell is $d_\alpha h_\beta l_\gamma$, where α, β, γ are running indices $\alpha = 1, \cdots, N_\alpha$; $\beta = 1, \cdots, N_\beta$; $\gamma = 1, \cdots, N_\gamma$ in the three orthogonal directions, respectively. The volume of the repeating cell is dhl where

$$d = \sum_{\alpha=1}^{N_\alpha} d_\alpha \quad , \quad h = \sum_{\beta=1}^{N_\beta} h_\beta \quad , \quad l = \sum_{\gamma=1}^{N_\gamma} l_\gamma \tag{9}$$

Any subcell can be filled in general by electro-magneto-thermo-elastic materials. Electro-magneto-thermo-elastic unidirectional long-fiber composites, short-fiber composites, porous materials, and laminated materials are obtained by a proper selection of the geometric dimensions of the subcells and with an appropriate material fillings.

The micromechanical model employs a first order expansion of the displacement in each subcell $(\alpha\beta\gamma)$ in terms of the local coordinates $(\bar{x}_1^{(\alpha)}, \bar{x}_2^{(\beta)}, \bar{x}_3^{(\gamma)})$ located at the center of the subcell.

$$u_i^{(\alpha\beta\gamma)} = w_i^{(\alpha\beta\gamma)} + \bar{x}_1^{(\alpha)} \, \phi_i^{(\alpha\beta\gamma)} + \bar{x}_2^{(\beta)} \, \chi_i^{(\alpha\beta\gamma)} + \bar{x}_3^{(\gamma)} \, \psi_i^{(\alpha\beta\gamma)} \qquad i = 1, 2, 3 \tag{10}$$

where $w_i^{(\alpha\beta\gamma)}$ are the displacement components at the center of the subcell, and $\phi_i^{(\alpha\beta\gamma)}$, $\chi_i^{(\alpha\beta\gamma)}$ and $\psi_i^{(\alpha\beta\gamma)}$ are the microvariables that characterize the linear dependence of the displacement $u_i^{(\alpha\beta\gamma)}$ on the local coordinates $\bar{x}_1^{(\alpha)}, \bar{x}_2^{(\beta)}, \bar{x}_3^{(\gamma)}$. In eqn.(2) and the sequel, repeated Greek letters do not imply summation.

The electric potential $\xi^{(\alpha\beta\gamma)}$ is also expanded linearly in terms of the local coordinates of the subcell:

$$\xi^{(\alpha\beta\gamma)} = \xi_0^{(\alpha\beta\gamma)} + \bar{x}_1^{(\alpha)} \, \xi_1^{(\alpha\beta\gamma)} + \bar{x}_2^{(\beta)} \, \xi_2^{(\alpha\beta\gamma)} + \bar{x}_3^{(\gamma)} \, \xi_3^{(\alpha\beta\gamma)} \tag{11}$$

Similarly, the magnetic potential $\eta^{(\alpha\beta\gamma)}$ is expanded linearly in terms of the local coordinates of the subcell:

$$\eta^{(\alpha\beta\gamma)} = \eta_0^{(\alpha\beta\gamma)} + \bar{x}_1^{(\alpha)} \, \eta_1^{(\alpha\beta\gamma)} + \bar{x}_2^{(\beta)} \, \eta_2^{(\alpha\beta\gamma)} + \bar{x}_3^{(\gamma)} \, \eta_3^{(\alpha\beta\gamma)} \tag{12}$$

The components of the small strain tensor $\epsilon_{ij}^{(\alpha\beta\gamma)}$ are given by

$$\epsilon_{ij}^{(\alpha\beta\gamma)} = \frac{1}{2}(\partial_i u_j^{(\alpha\beta\gamma)} + \partial_j u_i^{(\alpha\beta\gamma)}) \qquad i,j = 1,2,3 \tag{13}$$

where $\partial_1 = \partial/\partial \bar{x}_1^{(\alpha)}$, $\partial_2 = \partial/\partial \bar{x}_2^{(\beta)}$ and $\partial_3 = \partial/\partial \bar{x}_3^{(\gamma)}$.

The components of the electrical field $E_i^{(\alpha\beta\gamma)}$ in the subcell are obtained from the electric potential $\xi^{(\alpha\beta\gamma)}$ via

$$E_i^{(\alpha\beta\gamma)} = -\partial_i\, \xi^{(\alpha\beta\gamma)} \tag{14}$$

The components of the magnetic field $H_i^{(\alpha\beta\gamma)}$ in the subcell are obtained from the magnetic potential $\eta^{(\alpha\beta\gamma)}$ via

$$H_i^{(\alpha\beta\gamma)} = -\partial_i\, \eta^{(\alpha\beta\gamma)} \tag{15}$$

With the linear expansions of the displacements and the electric and magnetic potentials given by eqns.(10)-(12) the static equilibrium of the material within the subcell $(\alpha\beta\gamma)$ is satisfied, namely

$$\sigma_{ij,j}^{(\alpha\beta\gamma)} = 0 \tag{16}$$

Furthermore, in the absence of volume charges:

$$D_{i,i}^{(\alpha\beta\gamma)} = 0 \tag{17}$$

is satisfied, as well as

$$B_{i,i}^{(\alpha\beta\gamma)} = 0 \tag{18}$$

The volume average of the stresses $\bar{\sigma}_{ij}$, electric displacements $\bar{D}_i$ and the magnetic flux density $\bar{B}_i$ in the entire repeating cell (namely in the composite) is given by

$$\bar{\mathbf{Y}} = \frac{1}{dhl} \sum_{\alpha=1}^{N_\alpha} \sum_{\beta=1}^{N_\beta} \sum_{\gamma=1}^{N_\gamma} d_\alpha\, h_\beta\, l_\gamma\, \mathbf{Y}^{(\alpha\beta\gamma)} \tag{19}$$

Similarly, the volume average of the strains $\bar{\epsilon}_{ij}$, electric field components $\bar{E}_i$ and magnetic field components $\bar{H}_i$ in the composite is given by

$$\bar{\mathbf{X}} = \frac{1}{dhl} \sum_{\alpha=1}^{N_\alpha} \sum_{\beta=1}^{N_\beta} \sum_{\gamma=1}^{N_\gamma} d_\alpha\, h_\beta\, l_\gamma\, \mathbf{X}^{(\alpha\beta\gamma)} \tag{20}$$

In the framework of the GMC micromechanical model it is possible to establish a relationship between these two volume averages. For a piezoelectric composite such a relationship has been obtained (Aboudi, 1998) by establishing the appropriate concentration

matrix that relates the electromechanical field in the subcell to the composite strain $\bar{\epsilon}_{ij}$. The derivation of the concentration matrix is based on the application of the appropriate interfacial conditions between neighboring subcells as well as between repeating cells (to insure periodicity). In the present electro-magneto-thermo-elastic analysis these interfacial conditions are: (1) continuity of displacements, (2) continuity of tractions, (3) continuity of the electric potential, (4) continuity of normal electric displacements, (5) continuity of the magnetic potential, and, (6) continuity of the normal magnetic flux density. By generalizing the isothermal analysis (with $\Delta T = 0$) that was implemented in the piezoelectric case to the present electro-magneto-elastic composite, one can establishes the following relation between the local electro-magneto-elastic field in the subcell $\mathbf{X}^{(\alpha\beta\gamma)}$ and the average external (composite) macro field $\bar{\mathbf{X}}$ in the form

$$\mathbf{X}^{(\alpha\beta\gamma)} = \mathbf{A}^{(\alpha\beta\gamma)}\, \bar{\mathbf{X}} \tag{21}$$

where $\mathbf{A}^{(\alpha\beta\gamma)}$ is the electro-magneto-elastic concentration matrix associated with subcell $(\alpha\beta\gamma)$.

Substitution of eqn.(21) into (6) (with $\Delta T = 0$) yields

$$\mathbf{Y}^{(\alpha\beta\gamma)} = \mathbf{Z}^{(\alpha\beta\gamma)}\mathbf{A}^{(\alpha\beta\gamma)}\, \bar{\mathbf{X}} \tag{22}$$

Consequently, in conjunction with the averaging procedure given by (19), the following effective isothermal constitutive relations of the electro-magneto-elastic composite can be established

$$\bar{\mathbf{Y}} = \mathbf{Z}^*\, \bar{\mathbf{X}} \tag{23}$$

where the effective elastic stiffness, piezoelectric, piezomagnetic, dielectric, magnetic permeability and electromagnetic coefficients matrix $\mathbf{Z}^*$ of the composite is given by

$$\mathbf{Z}^* = \frac{1}{dhl} \sum_{\alpha=1}^{N_\alpha} \sum_{\beta=1}^{N_\beta} \sum_{\gamma=1}^{N_\gamma} d_\alpha\, h_\beta\, l_\gamma\, \mathbf{Z}^{(\alpha\beta\gamma)}\, \mathbf{A}^{(\alpha\beta\gamma)} \tag{24}$$

The structure of the square 12th order symmetric matrix $\mathbf{Z}^*$ is of the form

$$\mathbf{Z}^* = \begin{bmatrix} \mathbf{C}^* & \mathbf{e}^{*t} & \mathbf{q}^{*t} \\ \mathbf{e}^* & -\boldsymbol{\kappa}^* & -\mathbf{a}^* \\ \mathbf{q}^* & -\mathbf{a}^* & -\boldsymbol{\mu}^* \end{bmatrix} \tag{25}$$

where $\mathbf{C}^*$, $\mathbf{e}^*$, $\mathbf{q}^*$, $\boldsymbol{\kappa}^*$, $\mathbf{a}^*$ and $\boldsymbol{\mu}^*$ are the effective elastic stiffness, piezoelectric, piezomagnetic, dielectric, magnetic permeability and electromagnetic coefficients, respectively.

In order to incorporate the thermal effects in the composite, we utilize Levin's (1967) result to establish the effective thermal stress tensor, Λ_{ij}^* , pyroelectric coefficients, p_i^*, and pyromagnetic coefficients, m_i^*. This approach has been also followed by Dunn (1998) to establish the requested effective thermal moduli. To this end let us define the following vector of thermal stresses, pyroelectric and pyromagnetic coefficients material constants in the subcell $(\alpha\beta\gamma)$:

$$\boldsymbol{\Gamma}^{(\alpha\beta\gamma)} = [\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, \Lambda_6, p_1, p_2, p_3, m_1, m_2, m_3]^{(\alpha\beta\gamma)} \tag{26}$$

The corresponding global (effective) vector is defined by

$$\boldsymbol{\Gamma}^* = [\Lambda_1^*, \Lambda_2^*, \Lambda_3^*\Lambda_4^*, \Lambda_5^*, \Lambda_6^*, p_1^*, p_2^*, p_3^*, m_1^*, m_2^*, m_3^*] \tag{27}$$

According to Levin's result, the relation between $\boldsymbol{\Gamma}^{(\alpha\beta\gamma)}$ and $\boldsymbol{\Gamma}^*$ is given in terms of the electro-magneto-mechanical concentration matrices $\mathbf{A}^{(\alpha\beta\gamma)}$ as follows.

$$\boldsymbol{\Gamma}^* = \frac{1}{dhl} \sum_{\alpha=1}^{N_\alpha} \sum_{\beta=1}^{N_\beta} \sum_{\gamma=1}^{N_\gamma} d_\alpha\, h_\beta\, l_\gamma\, \mathbf{A}^{t(\alpha\beta\gamma)}\, \boldsymbol{\Gamma}^{(\alpha\beta\gamma)} \tag{28}$$

where $\mathbf{A}^{t(\alpha\beta\gamma)}$ is the transpose of $\mathbf{A}^{(\alpha\beta\gamma)}$. The above relation provides the effective thermal stress $\boldsymbol{\Lambda}^*$, pyroelectric $\mathbf{p}^*$ and pyromagnetic $\mathbf{m}^*$ vectors of the composite.

Consequently, the final anisothermal micromechanically established constitutive law of the electro-magneto-thermo-elastic multiphase composite is given by

$$\bar{\mathbf{Y}} = \mathbf{Z}^*\, \bar{\mathbf{X}} - \boldsymbol{\Gamma}^*\, \Delta T \tag{29}$$

Finally, the effective coefficients of thermal expansion α_i^* and the associated pyroelectric constants P_i^* and the pyromagnetic constants M_i^* of the composite can be assembled into the vector:

$$\boldsymbol{\Omega}^* = [\alpha_1^*, \alpha_2^*, \alpha_3^*, \alpha_4^*, \alpha_5^*, \alpha_6^*, P_1^*, P_2^*, P_3^*, M_1^*, M_2^*, M_3^*] \tag{30}$$

This vector is given by:

$$\boldsymbol{\Omega}^* = \mathbf{Z}^{*-1}\boldsymbol{\Gamma}^* \tag{31}$$

It should be mentioned that a similar relation holds for the local quantities, namely for the the coefficients of thermal expansion and the associated pyroelectric and pyromagnetic constants of the materials filling the subcells. In such a case $\mathbf{Z}^*$ and $\mathbf{\Gamma}^*$ should be replaced by $\mathbf{Z}^{(\alpha\beta\gamma)}$ and $\mathbf{\Gamma}^{(\alpha\beta\gamma)}$, respectively.

4. Applications

Following Li and Dunn (1998), let us consider a composite consisting of a $CoFe_2O_4$ piezo-magnetic matrix reinforced by $BaTiO_3$ piezoelectric material. Both phases are transversely isotropic with the axis of symmetry oriented in the 3-direction. The independent material constants of these constituents are given in Tables 1-3 (Li and Dunn, 1998). It should be noted that in both materials the electromagnetic coefficients are zero, i.e., $a_{ij} = 0$.

Table 1. Elastic material properties.

Material	$C_{11}(GPa)$	$C_{12}(GPa)$	$C_{13}(GPa)$	$C_{33}(GPa)$	$C_{44}(GPa)$
$BaTiO_3$	166	77	78	162	43
$CoFe_2O_4$	286	173	170	269.5	45.3

Table 2. Electric material properties.

Material	$e_{15}(C/m^2)$	$e_{31}(C/m^2)$	$e_{33}(C/m^2)$	$\kappa_{11}(10^{-9}C/Vm)$	$\kappa_{33}(10^{-9}C/Vm)$
$BaTiO_3$	11.6	-4.4	18.6	11.2	12.6
$CoFe_2O_4$	0	0	0	0.08	0.093

Table 3. Magnetic material properties.

Material	$q_{15}(N/Am)$	$q_{31}(N/Am)$	$q_{33}(N/Am)$	$\mu_{11}(10^{-6}Ns^2/C^2)$	$\mu_{33}(10^{-6}Ns^2/C^2)$
$BaTiO_3$	0	0	0	5	10
$CoFe_2O_4$	550	580.3	699.7	-590	157

Consider first a fibrous composite in which the $BaTiO_3$ continuous fibers are oriented in the 3-direction. The volume fraction of the fibers is denoted by v_f. In this case the GMC approach can be employed to generate the effective elastic, dielectric, magnetic permeability,

piezoelectric, piezomagnatic, and electromagnetic coupling moduli of the fibrous composite for $0 \leq v_f \leq 1$. To this end, the composite is modeled by the simplest configuration in which two subcells exist in every one of the orthogonal directions, see Fig. 1. These moduli were also generated by Li and Dunn (1998) who employed the micromechanical model of Mori-Tanaka (1993), MT. Figures 2 through 7 show a comparison between these effective moduli predicted by GMC and MT. It is readily seen that in some cases the predictions of both methods coincide, while in other cases some deviations are observed. It should be noted that both GMC and MT are approximate models (for some shortcomings of MT model, see Ferrari (1991)), and it is not possible to determine which one is more accurate.

Li and Dunn (1998) also generated the effective moduli of a bilaminated composite that consists of alternate layers of $BaTiO_3$ and $CoFe_2O_4$, in which the layering is in the 3-direction. Figs. 8-13 exhibit the corresponding effective moduli for this type of composite as predicted by GMC and MT. In this case the predictions of the two methods coincide for all values of volume fraction v_f of the $BaTiO_3$ phase.

As mentioned by Li and Dunn (1998), the most interesting behavior is the overall electro-magnetic coupling effect that is present in the composite, but not in either of the individual phases. This effect is shown in Figs. 8 and 13 for fibrous and bilaminated composites, respectively. As discussed by these authors, the different order of magnitudes of the coefficients a_{11}^* and a_{33}^* exhibited in both figures is attributed to the difference in magnetic constants between the piezoelectric $BaTiO_3$ and piezomagnetic $CoFe_2O_4$ phases. Further discussion of the various effective moduli behavior has been given by Li and Dunn (1998).

5. Conclusions

The fully coupled electro-magneto-thermo-elastic constitutive law has been incorporated into the GMC approach, thus providing the ability to model advanced composites that are sensitive to electric and magnetic fields. Since the GMC is a multiphase micromechanical model, it can be easily employed to investigate a wide range of effects, as has been shown by several investigators. For example, one can investigate the effect of inelasticity of the host material, the effect of weak bonding between the phases, the effect of fiber distribution and

architecture on the overall response of composites, and since GMC is a constitutive law, it can be utilized by a structural analysis package to investigate the behavior structures composed of these corresponding composite materials. Now with the electro-magneto generalization to the GMC outlined, herein, an entirely new class of materials can be analyzed and designed with this powerful approach.

Acknowledgment

References

Aboudi, J. (1996) Micromechanical analysis of composites by the method of cells - Update. Appl. Mech. Rev. **49**, S83-S91.

Aboudi, J. (1998) Micromechanical prediction of the effective coefficients of thermo-piezoelectric multiphase composites. J. Intell. Material Systems & Structures **9**, 713-722.

Arnold, S.M., Bednarcyk, B.A., Wilt, T.E. and Trowbridge, D. (1999) Micromechanics Analysis Code with Generalized Method of Cells (MAC/GMC) User Guide: version 3.0. NASA TM **1999-209070**.

Carman, G.P., Cheung, K.S. and Wang, D. (1995) Micro-mechanical model of a composite containing a conservative nonlinear electro-magneto-thermo-mechanical material. J. Intell. Material Systems & Structures **6**, 691-699.

Dunn, M.L. (1993) Micromechanics of coupled electroelastic composites: effective thermal expansion and pyroelectric coefficients. J. Appl. Physics **73**, 5131-5140.

Dunn, M.L. and Taya M. (1993) Micromechanics prediction of the effective electroelastic moduli of piezoelectric composites. Int. J. Solids & Structures **30**, 161-175.

Ferrari, M. (1991) Asymmetry and the high concentration limit of the Mori-Tanaka effective medium theory. Mech. Materials **11**, 251-256.

Herakovich, C.T. (1998) Mechanics of Fibrous Composites. Wiley, New York.

Levin, V.M. (1967) On the coefficients of thermal expansion of heterogeneous materials. Mech. Solids **2**, 58-61.

Li, J.Y. and Dunn, M.L. (1998) Micromechanics of magnetoelectroelastic composite materials: average field and effective behavior. J. Intell. Material Systems & Structures **9**, 404-416.

Mori, T., and Tanaka, K. (1973), Average stresses in matrix and average energy of materials with misfitting inclusions, Acta Metall. **21**, 571-574.

Nye, J.F. (1957) Physical Properties of Crystals. Oxford University Press.

Uchino, K. (1997) Piezoelectric Actuators and Ultrasonic Motors. Kluwer, Boston.
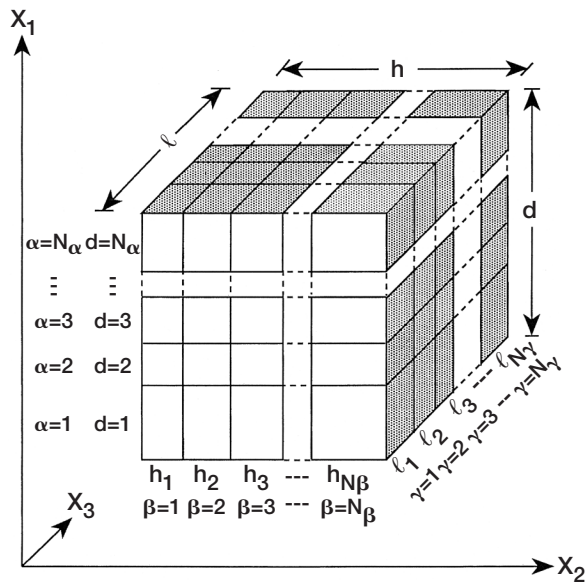
Figure 1.—A repeating cell in GMC consisting of N_α, N_β and N_γ subcells in the 1, 2 and 3 directions, respectively.
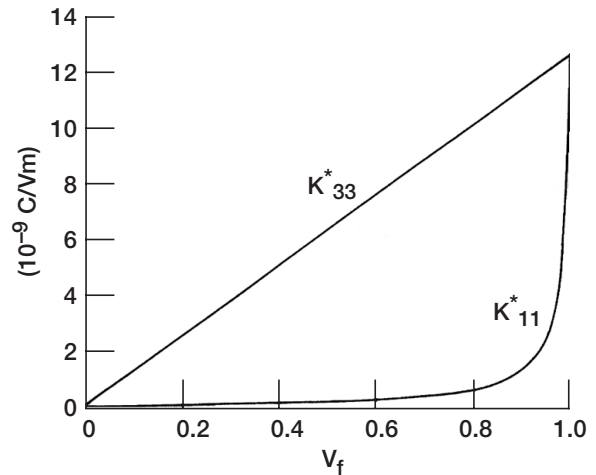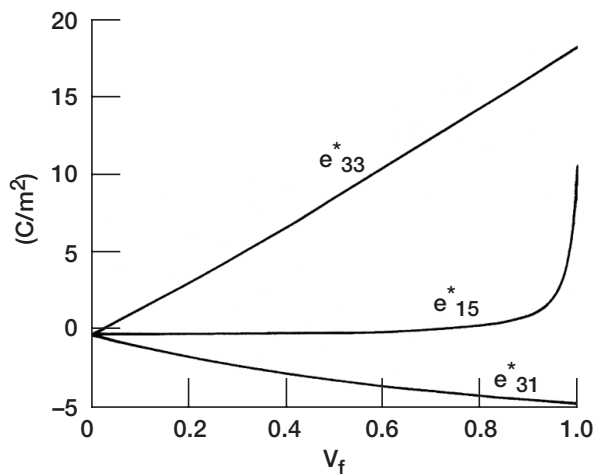

Figure 3.—Effective dielectric moduli of fibrous composite against the volume fraction of *BaTiO₃*. The predictions of GMC and MT coincide.
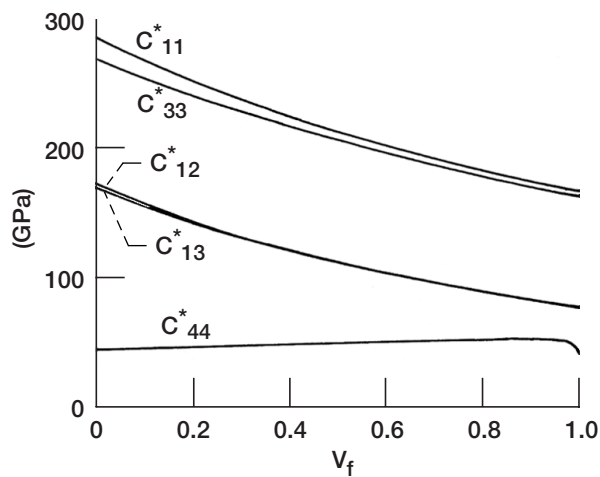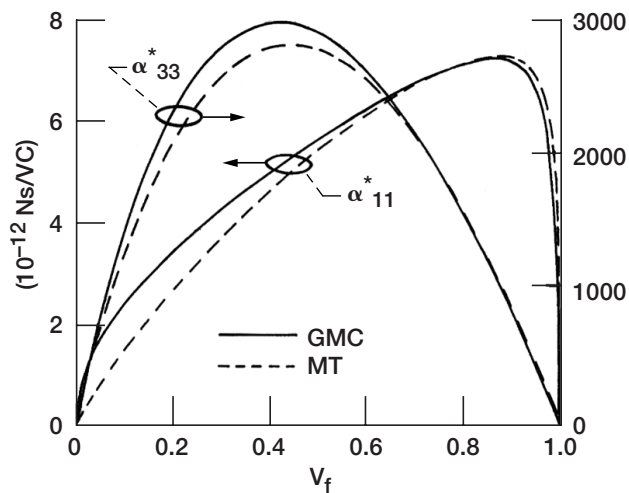

Figure 2.—Effective elastic moduli of fibrous composite against the volume fraction of *BaTiO₃*. The predictions of GMC and MT coincide.
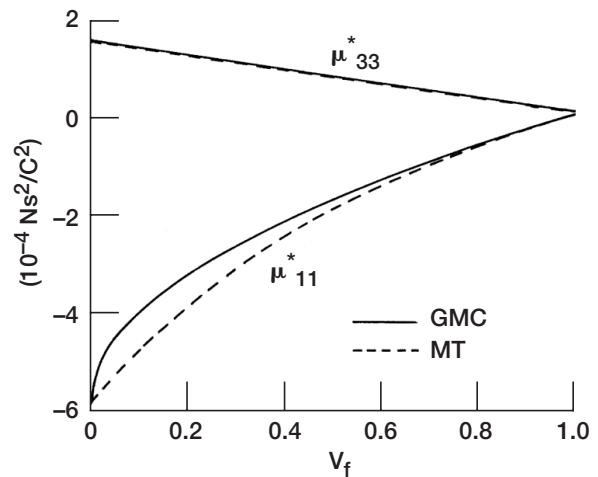
Figure 4.—Comparison between GMC and MT prediction of the effective magnetic permeability moduli of fibrous composite against the volume fraction of *BaTiO₃*.

Figure 5.—Effective piezoelectric moduli of fibrous
composite against the volume fraction of *BaTiO₃*.
The predictions of GMS and MT coincide.

Figure 7.—Comparison between GMC and MT predic-
tion of the effective electromagnetic coupling moduli
of fibrous composite against the volume fraction of
BaTiO₃.
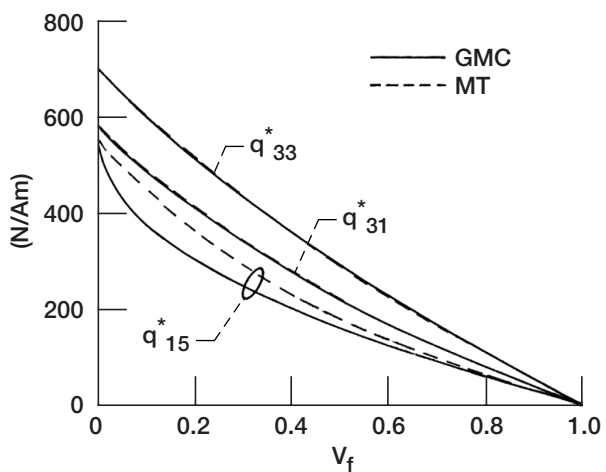
Figure 6.—Comparison between GMC and MT predic-
tion of the effective piezomagnetic moduli of fibrous
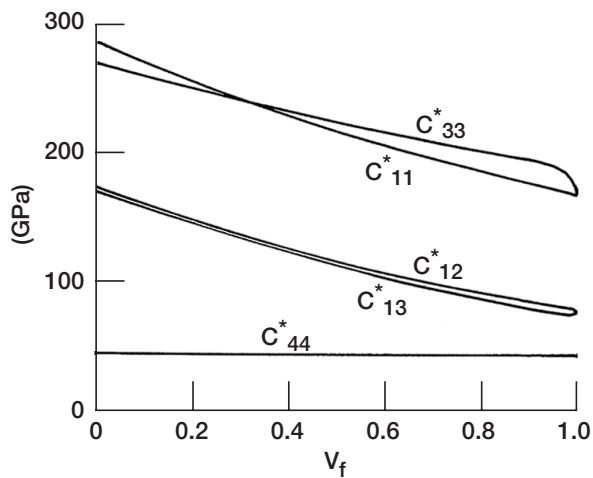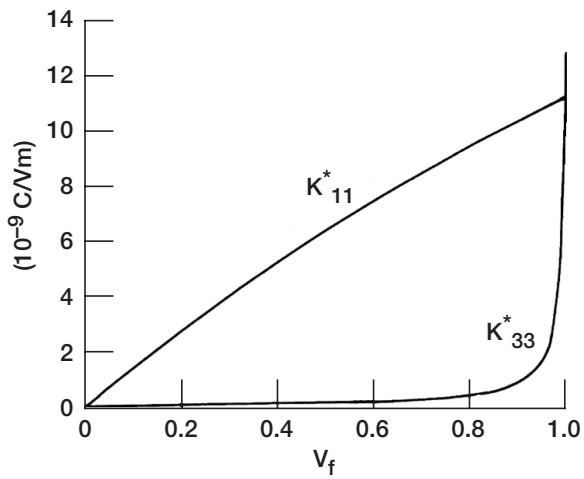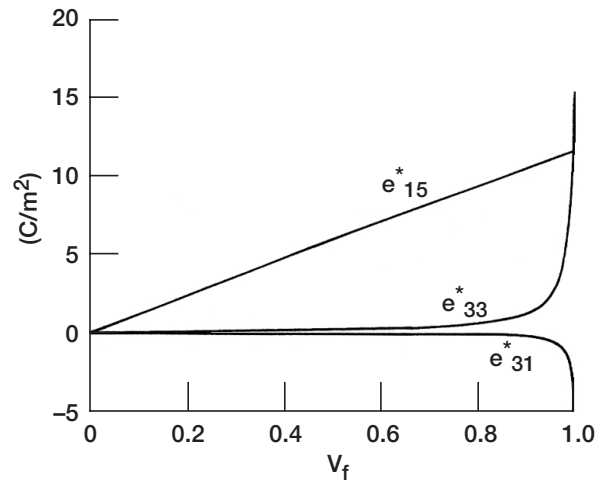composite against the volume fraction of *BaTiO₃*.

Figure 8.—Effective elastic moduli of bilaminated
composite against the volume fraction of *BaTiO₃*.
The predictions of GMC and MT coincide.

Figure 9.—Effective dielectric moduli of bilaminated composite against the volume fraction of *BaTiO₃*. The predictions of GMC and MT coincide.

Figure 11.—Effective piezoelectric moduli of bilaminated composite against the volume fraction of *BaTiO₃*. The predictions of GMC and MT coincide.
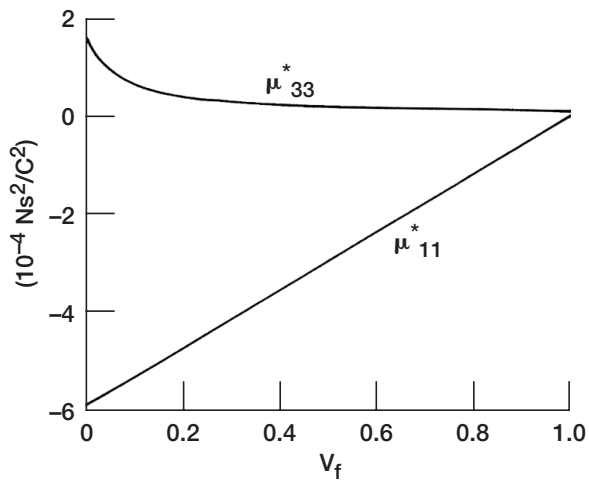

Figure 10.—Effective magnetic permeability moduli of bilaminated composite against the volume fraction of *BaTiO₃*. The predictions of GMC and MT coincide.
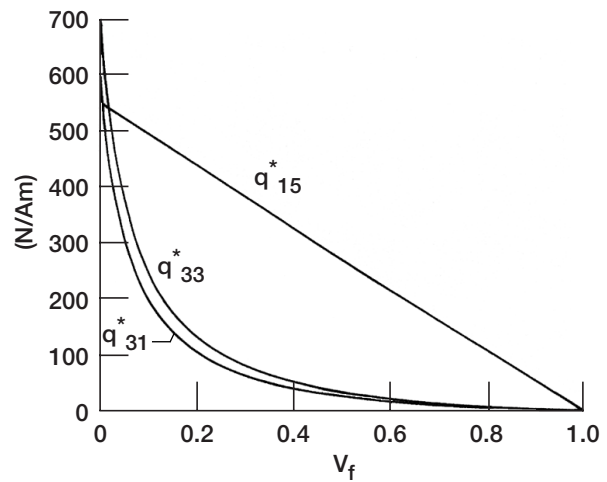

Figure 12.—Effective piezomagnetic moduli of bilaminated composite against the volume fraction of *BaTiO₃*. The predictions of GMC and MT coincide.
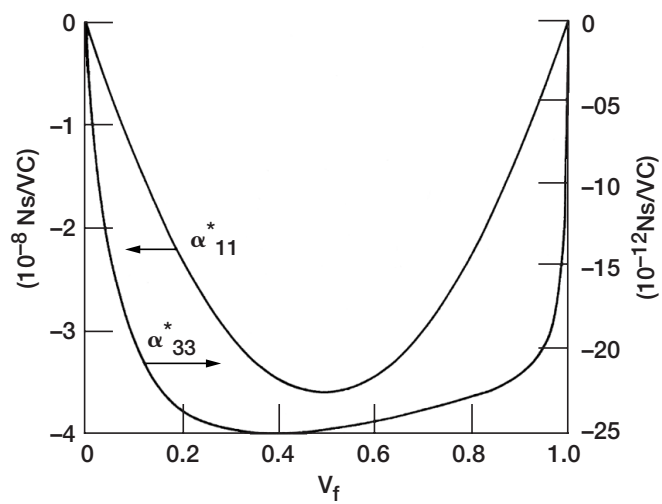
Figure 13.—Effective electromagnetic coupling moduli of bilaminated composite against the volume fraction of *BaTiO₃*. The predictions of GMC and MT coincide.

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The micromechanical generalized method of cells model is employed for the prediction of the effective moduli of electro-magneto-thermo-elastic composites. These include the effective elastic, piezoelectric, piezomagnetic, dielectric, magnetic permeability, electromagnetic coupling moduli, as well as the effective thermal expansion coefficients and the associated pyroelectric and pyromagnetic constants. Results are given for fibrous and periodically bilaminated composites.

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